MATERIAL TRANSPORT IN THE FREE VOLUME OF A FIXED GRANULAR BED

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The hydrodynamic state in the free volume of a granular bed during fluid flow through it is investigated. A mathematical model is proposed to describe material transport. Its parameters are determined experimentally.

The process of material transport in the free volume of a granular bed is determined to a considerable extent by the conditions for fluid flow in the space between granules.

The widely known models – diffusion [1-3], cellular [4, 5], with stagnation zones [6-9], and others – are insufficiently justified hydrodynamically and experimentally and are most often selected intuitively.

Selection and Justification of Model. To understand the phenomena occurring in the free volume of a bed and to facilitate the construction of a mathematical model, an experimental study of fluid flow in the space between granules was undertaken. The study was carried out in a flume with plane and three-dimensional models of a bed. In the work with a flume and plane model, we studied the flow around various groups of cylinders which represented the most characteristic portions of a bed cut by two planes. The three-dimensional model consisted of randomly arranged spheres 25 mm in diameter in a column 150 × 150 mm in cross section. The spheres were made of thin glass and were filled with water to reduce optical distortion. Some of the spheres were cut and fastened to the walls of the column. In all the experiments, the tracer was an aqueous solution of the ethyl ester of diethylamino-o-carboxyphenylxanthyl chloride, the distribution of which in the spaces between the granules was characterized by a greenish-yellow luminosity when the geometrical model was illuminated with ultraviolet light. In this way, the local hydrodynamic state around bed elements was observed. During an experiment, the flow rate was varied so that Re = 10-1000. It was established that the free volume of the bed was nonuniform over the entire range of variation of the Reynolds number; there were two regions, a flow zone (FZ) consisting of fluid streams in the space between the granules and a stagnation zone (SZ) located in the neighborhood of the points of contact between bed particles. The nature of material transport processes in the SZ and exchange with the FZ depend strongly on flow conditions in the external flow and on the Reynolds number. Qualitatively, one can distinguish three very typical regions. Thus, when $Re < 100$, the diffusion mechanism for material transport predominates, there is no marked vortex formation in the SZ, and its lightening or darkening occurs gradually. When $Re \approx 100$, vortices are formed in the outlet of the SZ and the zone is divided into two parts in this case, a diffusion region and a vortex region. As Re increases, the volume of the vortex region grows and intense pulsations of the boundary and volume of the SZ begin. When $Re \geqslant 800$, a vortex occupies practically the entire volume and predominates over the purely convective mechanism for material transport with the experimentally determined Strouhal number $\omega d/U$ being ≈ 0.5-0.6.

The results make it possible to assert that nonstationary mass transport in the free volume of a bed can be described by the following mathematical model:

$$(1-\alpha)\frac{\partial C_1}{\partial \tau} = \frac{1}{\mathrm{Pe}_1}\frac{\partial^2 C_1}{\partial \xi^2} - \frac{\partial C_1}{\partial \xi} - \beta\alpha (C_1 - C_2), \quad (1)$$

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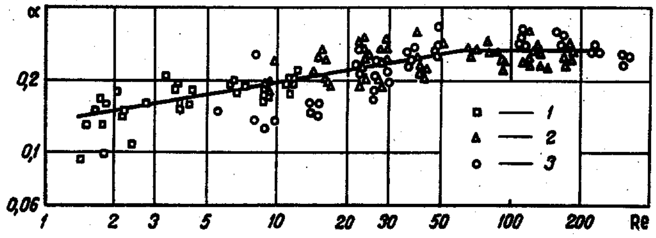

Fig. 1. Stagnant zone fraction in a granular bed: spheres with 1) d = 1.88 mm, 2) 5 mm, 3) 8 mm.

$$\varphi \frac{\partial C_2}{\partial \tau} = \beta (C_1 - C_2) - N \frac{\partial C_3}{\partial y}\bigg|_{y=l-\varphi}, \tag{2}$$

$$\frac{\partial C_3}{\partial \tau} = N \frac{\partial^2 C_3}{\partial y^2}, \tag{3}$$

where $\mathrm{Pe}_1 = \frac{Ud}{D_1}$; $N = \frac{D}{Ud}\left(\frac{d}{\delta}\right)^2$; $\tau = \frac{Ut}{d}$;

$$\varphi = \frac{\varepsilon_2'}{\varepsilon_2};\ \alpha = \frac{\varepsilon_2}{\varepsilon};\ \beta = \frac{\bar{\beta} d}{U};\ \xi = \frac{l}{d};\ y = \frac{r}{d}.$$

It should be noted that for gas motion through a fixed granular bed even for Re $\geqslant$ 20, the SZ is close to the volume for ideal mixing [10]; this difference in the nature of transport processes in the SZ for gases and liquids at identical values of the Reynolds number is explained by the differing degree of turbulence in the incoming flow. Initial perturbations having the same order of magnitude in the two cases die out in a shorter length of bed with a liquid because of its lower velocity. Thus by setting up a plane geometric model of a turbulation at the entrance, we managed to shift the region for the formation of pulsations in the SZ to Re $\approx$ 10-20.

If Re < 100, $\varphi = 0$ and the mathematical model (1)-(3) becomes similar to the Turner model [7]. Its basic parameters are: SZ fraction in the free volume of the bed (α); the parameter N, which determines mass transport in the stagnant zones; the quantity Pe_1, which characterizes longitudinal mixing in the FZ.

Experimental Determination of the SZ Fraction. The stagnant zone fraction was determined by the "tracer" method [10] on an experimental device similar to that described in [11]. The studies were made with glass spheres having d = 2, 5, 8 mm. The velocity of the ascending water flow was varied so that Re = 1-300. A KCl solution served as a tracer in all experiments. Measurement of the tracer penetration time was accomplished with platinum conductivity sensors set up in different sections of the bed and connected to ÉMP electronic bridges.

Results of the experimental determination of α are shown in Fig. 1. As indicated by the figure, there are two regions in the dependence of the SZ fraction on Reynolds number. Thus α increases when Re < 70 and is practically constant for Re = 70-250. In the analysis of the results for the purpose of obtaining criterial relationships, the following forms were chosen:

$$\text{for}\ \mathrm{Re} < 70\quad \alpha = A\mathrm{Re}^c, \tag{4}$$

$$\text{for}\ 70 < \mathrm{Re} < 250\quad \alpha = B. \tag{5}$$

Analysis of the experimental data was done on a "Minsk-2" computer using least squares. It turned out that A = 0.135, C = 0.16, and B = 0.27. The confidence intervals for the variation of these coefficients for a confidence factor P = 0.9 were ΔA = 0.015, ΔC = 0.02, and ΔB = 0.008.

Investigation of Mass Exchange between Zones and Longitudinal Mixing in the Flow Zone. The parameters N and Pe_1 were determined on the same experimental apparatus and under the same conditions as was the SZ fraction. The following analysis was performed to choose the location of the sensors in the bed. The equation system (1)-(3) was solved in the form of a Laplace transform including the boundary conditions given in [11]. Then, by variation of experimental conditions on the "Minsk-2" computer, those sensor coordinates were selected where the effect of transport processes in the inlet and outlet of the device on the solution was negligibly small (no more than 1-2%). It turned out that for a bed length L = 1.5 m and a velocity U = 0.3 m/sec it was advisable to set up the first sensor at a distance l_1 = 0.3 m from the entrance and the second at l_2 = 1.2 m. The results were then checked experimentally by varying the distance between the sensors. Thus the boundary conditions [11] can be simplified and presented in the form

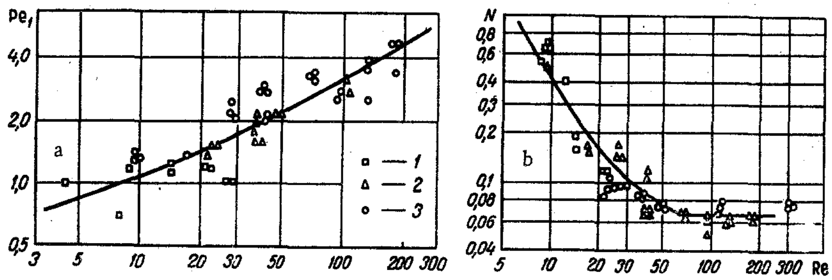

Fig. 2. a) Pe_1 as a function of Re, spheres with 1) d = 1.88 mm, 2) d = 8 mm, 3) d = 5 mm; the curve is the approximation given by Eq. (15). b) N as a function of Re, spheres with 1) d = 1.88 mm, 2) d = 5 mm, 3) d = 8 mm; curve is the approximation given by Eqs. (16) and (17).

$$\left.\begin{aligned} &\text{for } \xi = 0 \quad C_1(\xi) = C_0(t);\\ &\text{for } \xi \to \infty \quad \frac{\partial C_1}{\partial \xi} \to 0,\\ &\text{for } y = 0 \quad \frac{\partial C_3}{\partial y} = 0; \; y = 1 - \varphi \quad C_2 = C_3. \end{aligned}\right\} \tag{6}$$

The solution of (1)-(3) with (6) taken into account has the form

$$W(S) = \exp\left\{\frac{\mathrm{Pe}_1 \Delta\xi}{2}\left[1 - \sqrt{1 + \frac{4M}{\mathrm{Pe}_1}}\right]\right\},$$
$$M = (1-\alpha) S + \beta\alpha \left[1 - \frac{\beta}{\varphi S + \beta + \sqrt{SN}\,\mathrm{th}\sqrt{\frac{S}{N}}(1-\varphi)}\right], \tag{7}$$

as a Laplace transform where S is the Laplace variable.

One can obtain from Eq. (7), the amplitude-frequency characteristic

$$W(i\Omega) = A(\Omega)\exp[-i\varphi(\Omega)], \tag{8}$$

where $A(\Omega)$ and $\varphi(\Omega)$ are the amplitude- and phase-frequency characteristics (AFC and PFC). For Re < 100,

$$A(\Omega) = \exp\left\{\frac{\mathrm{Pe}_1 \Delta\xi}{2}\left[1 - \sqrt{\frac{\sqrt{R^2 + J^2}}{2} + \frac{R}{2}}\right]\right\}, \tag{9}$$

$$\varphi(\Omega) = \frac{\mathrm{Pe}_1 \Delta\xi}{2}\sqrt{\frac{\sqrt{R^2 + J^2}}{2} - \frac{R}{2}}, \tag{10}$$

$$R = 1 + \frac{4\alpha \mathrm{N} a}{\mathrm{Pe}_1}\,\frac{(\mathrm{th}\,a - \mathrm{tg}\,a) + \mathrm{tg}\,a \cdot \mathrm{th}\,a\,(\mathrm{th}\,a + \mathrm{tg}\,a)}{1 + (\mathrm{th}\,a \cdot \mathrm{tg}\,a)^2}; \tag{11}$$

$$J = (1-\alpha)\Omega + \alpha \mathrm{N} a\,\frac{(\mathrm{th}\,a + \mathrm{tg}\,a) - \mathrm{tg}\,a \cdot \mathrm{th}\,a\,(\mathrm{th}\,a - \mathrm{tg}\,a)}{1 + (\mathrm{th}\,a \cdot \mathrm{tg}\,a)^2}; \tag{12}$$

$$a = \sqrt{\frac{\Omega}{\mathrm{N}}}; \quad \Omega = \frac{\omega d}{U}.$$

The corresponding values of the AFC and PFC from the experimental curves were determined from

$$A_e(\Omega) = \frac{\sqrt{\left[\int_0^\infty C_2(\tau)\cos(\Omega\tau)\,d\tau\right]^2 + [C_2(\tau)\sin(\Omega\tau)\,d\tau]^2}}{\sqrt{\left[\int_0^\infty C_1(\tau)\cos(\Omega\tau)\,d\tau\right]^2 + [C_1(\tau)\sin(\Omega\tau)\,d\tau]^2}}, \tag{13}$$

$$\varphi_e(\Omega) = \mathrm{arctg}\frac{\int_0^\infty C_2(\tau)\sin(\Omega\tau)\,d\tau}{\int_0^\infty C_2(\tau)\cos(\Omega\tau)\,d\tau} - \mathrm{arctg}\frac{\int_0^\infty C_1(\tau)\sin(\Omega\tau)\,d\tau}{\int_0^\infty C_1(\tau)\cos(\Omega\tau)\,d\tau}, \tag{14}$$

where $C_1(\tau)$ and $C_2(\tau)$ are functions of the change in tracer concentration in the cross sections at l_1 and l_2. They were determined on the experimental device by conductivity sensors installed in the two sections of the bed when a pulse of tracer (KCl, $ZnSO_4$, or K_2SO_4) was fed into the inlet.

The parameters Pe_1 and N were selected on the "Minsk-2" computer by comparison of the theoretical values for the amplitude-frequency characteristics with the corresponding experimental values. To accomplish this, a grid of Pe_1 and N values was given and that set determined which provided a minimum squared deviation no more than 2%. It should be noted that over the entire range Pe_1 = 0.5-10 and N = 0.001-1.0 there was only a single set of parameters which best described the experimental data.

Results of the study of the longitudinal mixing coefficients (for Sc = 500 and KCl tracer) are shown in Fig. 2a; results for the parameter N are shown in Fig. 2b. To investigate the effect of the Schmidt number on mass transport, $ZnSO_4$ (Sc = 1180) and K_2SO_4 (Sc = 650) were used as tracers in addition to KCl. We found that $N \sim Sc^{-1.0}$ and $Pe_1 \sim Sc^{0.20}$.

Through analysis of the experimental data for longitudinal transport, the relation

$$Pe_1 = B_1 + A_1 Re^{C_1} Sc^{0.20}, \tag{15}$$

was selected. The quantity B_1 can be obtained from the following physical considerations: for Re < 1, the intensities of the transport processes in the longitudinal and transverse directions of the bed become commensurable. This leads to the fact that the free volume of the bed can be described by a diffusion model with Pe_1 equal to 0.5 [1, 2]. Thus B_1 = 0.5. A_1 and C_1, which are determined analogously to A and B, have the values A_1 = 0.042, C_1 = 0.67. The confidence intervals for P = 0.9 for their variations are ΔA_1 = 0.01 and ΔC_1 = 0.07.

Since $N = (D/Ud)(d/\delta)^2$, we obtain $NReSc = (d/\delta)^2$ by multiplying both sides by ReSc. As follows from the mathematical model assumed, $d/\delta \sim \alpha^{-1}$ and for Re < 70

$$N = B_2 + A_2 Re^{-1.32} Sc^{-1.0}. \tag{16}$$

As already noted in the first section, when Re > 100 a vortex is formed in the outlet of the SZ which intensifies mass exchange with the FZ. Then, considering D as some effective quantity proportional to linear velocity and granule diameter and assuming that d/δ = const (see Fig. 1), we obtain

$$N = B_3. \tag{17}$$

The coefficients B_2, A_2, and B_3 determined by the method described above prove to be B_2 = 0.04, A_2 = 3500, and B_3 = 0.065. The confidence intervals for their variation are ΔB_2 = 0.005, ΔA_2 = 600, and ΔB_3 = 0.005.

NOTATION

U	is the fluid velocity in free volume of bed; m/sec;
d	is the sphere diameter, m;
D_1	is the coefficient of longitudinal mixing in flow-through region, m^2/sec;
D	is the molecular diffusivity, m^2/sec;
δ	is the depth of stagnation zone, m;
t	is the time, sec;
ε_1	is the fraction of flow-through zones in a bed;
ε_2'	is the fraction of vortex zone;
ε	is the bed porosity;
l	is the longitudinal coordinate, m;
$\bar{\beta}$	is the mass transfer between zones, m/sec;
C_1, C_2, C_3	are the tracer concentrations in flow-through, vortex, and diffusional regions, respectively, fractions;
ω	is the frequency of vortex fluctuations in flow zone, sec^{-1};
r	is the coordinate along depth of stagnation zone, m;
$\Delta\xi$	is the distance between probes;
ν	is the kinematic viscosity, m^2/sec;
$Re = Ud/\nu$	is the Reynolds number;
Sc	is the Schmidt number.

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